High-Energy Diffraction Scattering*

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The experimental data of Foley *et al.* on elastic proton-proton scattering have been analyzed in a phenomenological manner using the impact-parameter representation for the scattering amplitude in the form given by Blankenbecler and Goldberger. The detailed shape and energy dependence of the elastic differential cross section, the small ratio of the elastic-scattering cross section to the total cross section, and the constant total cross section can all be explained with remarkable ease using a smooth weight function with reasonable properties. Two cases were considered. In the first, the scattering amplitude was assumed to be purely imaginary. In order to account for the observed narrowing of the elastic diffraction peak, considered as a function of the square of the invariant momentum transfer t it is then necessary that the radius of the interaction region increase, and its opacity decrease, with increasing energy. The necessary energy dependence cannot be determined uniquely, but is consistent with that expected on the basis of semiclassical considerations. However, some unexpected difficulties are encountered at the lower momenta, and evidence is adduced for the existence of a significant real part of the scattering amplitude for moderate values of $|t|$. The consequences of a nonzero real amplitude were investigated in a second model. It was found that the shrinking of the diffraction peak can be explained using real and imaginary amplitudes with fixed t dependence, with the magnitude of the former decreasing relative to that of the latter with the energy dependence expected on the basis of potential-scattering considerations. It is therefore not possible to conclude from the observed shrinking of the diffraction peak that a Regge pole mechanism is operative, or even that the effective radius of the absorptive region is increasing, without first establishing experimentally the behavior of the real part of the scattering amplitude for $t<0$. The observation that $|\mathrm{Re}f(s,0)/\mathrm{Im}f(s,0)|^2 \ll 1$, as established using the optical theorem, is insufficient for this purpose. The $\pi^- - p$ elastic scattering data of Foley *et al.* have also been analyzed.

I. INTRODUCTION

THE hypothesis that the behavior of elastic scattering amplitudes is determined for high energies and small momentum transfers by one, or at most a few, Regge poles, leads to the expectation that the diffraction peak characteristic of scattering in this regime will become narrower as a function of momentum transfer for increasing energies.[1] This expectation is not fulfilled by recent data on elastic π-p scattering,[2,3] nor is it clear that the observed narrowing of the diffraction peak in p-p scattering[3,4] is, in fact, attributable to a simple Regge pole mechanism. Current theoretical studies[5] suggest that the singularities of the scattering amplitude as a function of a complex angular momentum include cuts as well as isolated Regge poles. The effects of such cuts on the scattering amplitude are essentially unknown, but it is not unlikely that their presence requires some modification of the predictions of the simple Regge pole hypothesis, at least for energies in the range which is presently accessible.[6] We thought it of interest, therefore, to re-examine the present experimental data using a more conventional, albeit phenomenological, description of diffraction scattering to determine if such a description is reasonable, and to explore its consequences.[7] We have not examined the various Regge pole models in any detail. The main results of our analysis are the following:

The shape of the elastic p-p differential cross section, and the small ratio of the elastic-scattering cross section to the total cross section can be explained with remarkable ease using an impact parameter model with reasonable properties. The possible spin dependence of the scattering amplitude was ignored.

If it is assumed that the scattering amplitude is purely imaginary, the narrowing of the p-p diffraction peak between 7 and 26 BeV implies that the effective radius of the absorptive region increases with increasing energy. The constancy of the total cross section in this energy range requires a concomitant decrease in the opacity of that region.[1] A simple model in which these parameters approach constant values at very high energies provides an excellent fit to the data. However,

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[1] G. F. Chew and S. C. Frautschi, Phys. Rev. Letters **7**, 394 (1961). S. C. Frautschi, M. Gell-Mann, and F. Zachariasen, Phys. Rev. **126**, 2204 (1962). A comprehensive treatment of nucleon-nucleon scattering based on the Regge pole hypothesis is given by I. J. Muzinich, *ibid.* **130**, 1571 (1963).

[2] C. C. Ting, L. W. Jones, and M. L. Perl, Phys. Rev. Letters **9**, 468 (1962).

[3] K. J. Foley, S. J. Lindenbaum, W. A. Love, S. Ozaki, J. J. Russell, and L. C. L. Yuan, Phys. Rev. Letters **10**, 376 (1963).

[4] A. N. Diddens, E. Lillethun, G. Manning, A. E. Taylor, T. G. Walker, and A. M. Wetherell, Phys. Rev. Letters **9**, 108 and 111 (1962). G. Cocconi, A. N. Diddens, E. Lillethun, G. Manning, A. E. Taylor, T. G. Walker, and A. M. Wetherell, *ibid.* **7**, 450 (1961).

[5] For example, R. Newton (unpublished) claims to have found energy-dependent branch points in the complex angular momentum plane, while V. N. Gribov and I. Ya. Pomeranchuk [Phys. Letters **2**, 239 (1962)] have argued the existence of essential singularities.

[6] This problem has been explored to some extent by I. R. Gatland and J. W. Moffat, Phys. Rev. **129**, 2812 (1963); **132**, 442 (1963).

[7] A more restricted analysis of this sort using the eikonal approximation of potential scattering theory has been made by R. Serber, Phys. Rev. Letters **10**, 357 (1963). The possible connection of the shrinking of the p-p diffraction peak with the disappearance of the hard core in the p-p interaction has been investigated by G. E. Brown, in Proceedings of the International Conference on Nucleon Structure (to be published).

at the lowest energy used by Foley *et al.*[3] [6.8-BeV/c incident momentum], the opacity exceeds the limit implied by unitarity and the assumption of a purely imaginary scattering amplitude. The result is quite general, and suggests either that the real part of the scattering amplitude is of significant magnitude for large momentum transfers, contrary to our initial assumption, or that the spin dependence of the scattering amplitude cannot be ignored.

As an alternative suggested by the foregoing difficulty, we have considered a model in which the real part of the scattering amplitude is nonvanishing, but approaches zero relative to the imaginary part at high energies with the p^{-1} energy dependence expected from potential-scattering considerations. This model yields a good fit to the data including the narrowing of the diffraction peak even if the dependence of the real and imaginary parts of the scattering amplitude on the momentum transfer is independent of the energy. It is required only that the ratio $|\mathrm{Re}f(s,t)/\mathrm{Im}f(s,t)|$ increase with increasing $|t|$; in particular, $|\mathrm{Re}f(s,0)/\mathrm{Im}f(s,0)|$ need not be large. This model also has the attractive feature that a constant total cross section can be obtained with a fixed or increasing opacity. As is the case for the model with $\mathrm{Re}f(s,t)=0$, the diffraction peak ceases to shrink at very high energies, and σ_{el} approaches a constant value, $\sigma_{\mathrm{el}}/\sigma_T \approx \frac{1}{4}$. We have thus far not considered in detail the possible effects on the cross sections of a non-negligible spin dependence of the scattering amplitude, but it is probable that a satisfactory fit to $d\sigma_{\mathrm{el}}/dt$ and σ_T could be obtained with a much smaller real amplitude if an appropriate spin dependence was introduced.

It is clear from the foregoing results that the behavior of the total p-p scattering cross section, and the narrowing of the elastic-diffraction peak, while consistent with a simple Regge pole mechanism, are not necessarily evidence for its existence. It is especially important in this connection that experiments be conducted to determine the spin dependence and reality properties of the scattering amplitude.

The shape of the diffraction peak in π-p scattering, and the absence of any significant shrinkage between incident momenta of 7 and 17 BeV/c,[3] are readily explicable using a model very similar to that for p-p scattering, but with a purely imaginary scattering amplitude with fixed geometrical properties. No evidence can be adduced for the possible existence of a significant real part of the scattering amplitude; neither can such a real part by excluded.

II. ANALYSIS OF p-p AND π-p DIFFRACTION SCATTERING

A. The Impact-Parameter Formalism

We will neglect the effects of spin and write the [unsymmetrized[8]] amplitude for elastic p-p scattering in the form of an integral over impact parameters,[9,10]

$$f(s,t)=\int_0^\infty J_0(b(-t)^{1/2})H(b,s)bdb. \qquad (1)$$

[A similar expression may be obtained for the pion-proton scattering amplitude.] Here s is the square of the total energy in the center-of-mass system, and t is the square of the 4-momentum transfer, $-t=2p^2(1-\cos\theta)$, $4p^2=s-4m^2$. We will use a normalization for $f(s,t)$ such that the differential elastic scattering cross section is given by[11]

$$d\sigma_{\mathrm{el}}/d\Omega=|f(s,t)|^2. \qquad (2)$$

The total cross section is then given by the optical theorem,

$$\sigma_T/4\pi=p^{-1}\,\mathrm{Im}f(s,0)=p^{-1}\int_0^\infty \mathrm{Im}H(b,s)bdb. \qquad (3)$$

The properties of the weight function $H(b,s)$ have been discussed in detail by Blankenbecler and Goldberger.[9] We note only the following. At energies sufficiently high that $pb_{\max}\gg 1$, $H(b,s)$ is related to the usual partial-wave scattering amplitudes f_l by[12]

$$f_l(s)=(2ip)^{-1}(S_l-1)=p^{-1}\int_0^\infty J_{2l+1}(2pb)H(b,s)db \approx (2p^2)^{-1}H(b,s), \quad l\approx pb. \qquad (4)$$

Thus, denoting the complex phase shift for angular

[8] The proper symmetrization of the scattering amplitude can be accomplished by adding to the right-hand side of Eq. (1) a similar term in which t is replaced by $u=-2p^2(1+\cos\theta)$. The added term is of negligible magnitude in the region of the forward diffraction peak.

[9] R. Blankenbecler and M. L. Goldberger, Phys. Rev. **126**, 766 (1962), and references contained therein.

[10] The customary treatment of high-energy potential scattering in the eikonal approximation leads to an expression of the form given in Eq. (1), with $H(b,s)$ expressed in terms of the potential $V(r)$ by

$$H(b,s)\approx ip[1-e^{i\chi(b,s)}],$$

with

$$\chi(b,s)=(2p)^{-1}\int_{-\infty}^{\infty} dzV([b^2+z^2]^{1/2}).$$

[The derivation of this result is given, for example, by R. Glauber, *Lectures in Theoretical Physics* (Interscience Publishers, Inc., New York, 1958), Vol. I, p. 315.] This result does not appear to be especially appropriate for the present analysis. Not only is one forced to choose a potential, the existence of which is doubtful, but most simple choices lead to essential singularities of the scattering amplitude, as was noted in Ref. 9, and to the appearance of unwanted diffraction zeros in the scattering amplitude. The latter are a consequence, in general, of the very rapid variation of the exponential, and the resulting tendency of the weight function to display a sharp corner at that value of b for which $i\chi(b,s)\approx -1$. On the other hand, the general impact parameter formalism as developed in Ref. 9 follows directly from the [assumed] existence of a Mandelstam representation for the scattering amplitude. [See, however, Ref. 7.]

[11] This normalization differs from that of Ref. 9.

[12] One can obtain Eq. (1) directly as an approximation to the partial-wave expansion by replacing the Legendre polynomials by their asymptotic forms, $P_l(x)\approx J_0(b(-t)^{1/2})$, $(pb)^2=l(l+1)$, and changing the sum to an integral. See, for example, the discussion given by K. R. Greider and A. E. Glassgold, Ann. Phys. (N.Y.) **10**, 100 (1960), and the numerous references contained therein.

momentum l by $\delta_l=\alpha_l+i\beta_l$, $\beta_l\geq 0$,

$$p^{-1}\,\mathrm{Im}H(b,s)\approx 1-e^{-2\beta_l}+2e^{-2\beta_l}\sin^2\alpha_l \quad (5)$$

and

$$p^{-1}\,\mathrm{Re}H(b,s)\approx 2e^{-2\beta_l}\sin\alpha_l\cos\alpha_l. \quad (6)$$

Aside from its general analytic properties, there is relatively little more which can be determined about the structure of $H(b,s)$ unless one is willing to adopt a specific model for the interaction. However, for a scattering amplitude which satisfies a Mandelstam representation, $H(b,s)$ can be expressed as a continuous superposition of hyperbolic Bessel functions $K_0(\mu b)$,[13]

$$H(b,s)=\int_{\mu_0}^{\infty}\mathcal{H}(\mu,s)K_0(\mu b)d\mu\,, \quad (7)$$

where $\mathcal{H}(\mu,s)$ is related by kinematic factors to the Mandelstam weight function $A(s,t')$, and $t'=\mu^2$.[8] The continuous parameter μ may be regarded as the range parameter for an effective Yukawa potential with a strength which is proportional to $\mathcal{H}(\mu,s)$. The range of μ is restricted for both p-p and π-p scattering to $\mu\geq\mu_0=2m_\pi$. Consequently, $H(b,s)$ decays asymptotically at least as rapidly as $K_0(2m_\pi b)$ for $b\rightarrow\infty$,[14]

$$K_0(2m_\pi b)\rightarrow(\pi/4m_\pi b)^{1/2}e^{-2m_\pi b},\quad 2m_\pi b\gg 1. \quad (8)$$

We should note, however, the restrictions imposed on $H(b,s)$ by unitarity: for a predominantly absorptive interaction with small real phase shifts, Eqs. (5) and (6) imply that $p^{-1}\,\mathrm{Re}H(b,s)\approx 0$ and $p^{-1}\,\mathrm{Im}H(b,s)\approx[1-e^{-2\beta_l}]\leq 1$. If the range B of the absorptive region is defined as that value of b for which $p^{-1}\,\mathrm{Im}H(b,s)$ first drops significantly below the limit imposed by unitarity, it is clear that B may be larger than the nominal maximum $b_{\max}\approx(2m_\pi)^{-1}$ implied by Eq. (8), provided only that the effective interaction is sufficiently strong.

B. Proton-Proton Diffraction Scattering: General Considerations

The impact-parameter formalism has been applied in a straightforward manner to the analysis of the recent data on p-p scattering in the energy range of 7 to 26 BeV.[3,4] Basic to the method is the assumption that the weight function $H(b,s)$ varies smoothly with b and has a range which is large compared to p^{-1}, $pb_{\max}\gg 1$, or equivalently, that many partial waves contribute to the scattering, and that the partial-wave scattering amplitudes $f_l(s)$ are smooth functions of l. The possible functional forms of $H(b,s)$ are limited to some extent by the experimental results. We note in particular the following characteristics of p-p scattering for energies in the range under consideration. (1) The magnitude of the total scattering cross section is essentially independent of the energy, $\sigma_T\approx 40$ mb.[15] This result requires that the integral in Eq. (3) vary linearly with the momentum, but does not determine the energy dependence of the integrand $\mathrm{Im}H(b,s)$. Although a scattering amplitude dominated by the vacuum Regge pole would possess the requisite properties, this is by no means the only alternative. For example, it is sufficient to assume that, to a first approximation, $p^{-1}\,\mathrm{Im}H(b,s)$ is a function of b alone, that is, $\mathrm{Im}H(b,s)\approx ph(b)$.[16] (2) The differential elastic-scattering cross section decreases nearly exponentially as a function of t over two to three orders of magnitude, and displays no prominent diffraction minima. Furthermore, the ratio of the elastic-scattering cross section to the total cross section is quite small, $\sigma_{\mathrm{el}}/\sigma_T\lesssim 1/4$.[4] These results are readily explicable if $H(b,s)$ approaches zero smoothly for large b, with a range in b for the "surface thickness" which is comparable to the "radius" of the distribution.[17] (3) The extrapolated values of $d\sigma_{\mathrm{el}}/dt$ for $t=0$ are close at all energies to the lower limits imposed by the optical theorem, $[d\sigma_{\mathrm{el}}/dt]_{t=0}\approx\frac{1}{2}(\sigma_T/4\pi)^2$.[4] This suggests that the scattering amplitude is predominantly imaginary for $t=0$, but the bounds on the magnitude of $|\mathrm{Re}f(s,0)|$ are rather weak. No information concerning the real part of the scattering amplitude is available for $t<0$. (4) The width of the elastic diffraction peak considered as a function of t decreases markedly with increasing s.[3,4] We should note immediately that the model suggested by points (1) and (3), $\mathrm{Im}H(b,s)=ph(b)$, $\mathrm{Re}H(b,s)\approx 0$, is unable to account for this effect. The associated scattering amplitude is of the form $f(s,t)=ipg(t)$, and the shape of the diffraction peak is consequently independent of s. More generally, it is clear that an acceptable model for $H(b,s)$ must lead to a scattering amplitude which cannot be written as the product of a function of s and a function of t.[18]

We shall examine in the following paragraphs two models which account in a satisfactory manner for the foregoing phenomena, yet do not depart too drastically from the usual semiclassical ideas concerning diffraction scattering. Other models, based on the Regge pole hypothesis, have been considered by Foley *et al.*[3] and by Gatland and Moffat.[6]

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[13] G. N. Watson, *A Treatise on the Theory of Bessel Functions* (Cambridge University Press, New York, 1945), 2nd ed., p. 78.

[14] G. N. Watson, Ref. 13, p. 202.

[15] S. J. Lindenbaum, W. A. Love, J. A. Niederer, S. Ozaki, J. J. Russell, and L. C. L. Yuan, Phys. Rev. Letters **7**, 185 (1961).

[16] That this assumption can lead to difficulties with the joint requirements of unitarity and the analyticity implied by the Mandelstam representation, has been argued by V. N. Gribov, Nucl. Phys. **22**, 249 (1961), but the objection is probably not serious at the energies of present interest.

[17] The difficulties with the optical model cited by Lovelace [C. Lovelace, Nuovo Cimento **25**, 730 (1962)] arise from the use of a sharply delimited absorptive region [the "gray-disk" model], and are not relevant to diffuse surfaced models.

[18] Such a model is not, in general, subject to the Gribov "disease," Ref. 16.

C. Proton-Proton Scattering: Amplitudes with $\mathrm{Re}f(s,t)\approx 0$

We shall assume initially that the elastic-scattering amplitude is purely imaginary. The optical theorem, Eq. (3), and the constant total cross section σ_T restrict the area under the curve of $p^{-1}b\,\mathrm{Im}H(b,s)$ to a constant value. On the other hand, the narrowing of the diffraction peak with increasing s requires that the effective radius of $\mathrm{Im}H(b,s)$ as a function of b increase. These two requirements are consistent only if the magnitude of $\mathrm{Im}H(b,s)$ decreases as the radius is increased.[1] The increase in the radius of $\mathrm{Im}H(b,s)$ can be understood in terms of an increase in the effective strength of the long range part of the interaction with the opening of new channels, coupled with the upper bound on the magnitude of $\mathrm{Im}H(b,s)$ imposed by unitarity. The simultaneous decrease in the opacity of the interaction region is more difficult to interpret[19] [and is, in addition, somewhat unattractive from a semiclassical point of view]. The particular model which we have considered for the weight function $\mathrm{Im}H(b,s)$ was suggested by the general expression for $H(b,s)$ noted in Eq. (7), and consists of a single hyperbolic Bessel function with an argument modified so as to satisfy the unitarity restriction for all b,

$$\mathrm{Im}H(b,s)=p\gamma(s)K_0([\lambda^2+\mu^2b^2]^{1/2})/K_0(\lambda)\,. \qquad (9)$$

Here γ is an opacity factor which measures the extent to which the low partial waves are absorbed, $0\leq\gamma\leq 1$, while the energy-dependent parameters λ and μ determine the range and the asymptotic decay rate of $\mathrm{Im}H(b,s)$. The significance of these parameters may be seen more clearly if it is noted that, for $\lambda\gg 1$ and $\mu b\ll\lambda$,

$$\mathrm{Im}H(b,s)\approx p\gamma[1+(\mu b/\lambda)^2]^{-1/2}e^{-\mu^2b^2/2\lambda}. \qquad (10)$$

The weight function consequently has a nearly Gaussian dependence on b in this region, with a characteristic radius $B\sim(2\lambda/\mu^2)^{1/2}$. For $\mu b\gg\lambda$, the behavior is essentially that of $K_0(\mu b)$, Eq. (8), that is, the asymptotic behavior expected for a Yukawa potential of range μ^{-1}.[9] The scattering amplitude can be evaluated exactly,[20]

$$f(s,t)=ip\gamma\lambda\mu^{-2}[1-(t/\mu^2)]^{-1/2}\times K_1(\lambda[1-(t/\mu^2)]^{1/2})/K_0(\lambda)\,, \qquad (11)$$

and decreases exponentially in t for $|t|/\mu^2\ll 1$, and more slowly for $|t|/\mu^2\gtrsim 1$. Use of the optical theorem, Eq. (3), and the asymptotic expansions of the Bessel functions for large λ,[14] leads to a simple expression for the total cross section,

$$\sigma_T=4\pi\gamma\lambda\mu^{-2}K_1(\lambda)/K_0(\lambda)\to 4\pi\gamma\frac{\lambda}{\mu^2}\times\left[1+\frac{1}{2\lambda}-\frac{1}{8\lambda^2}+\cdots\right],\ \lambda\gg 1\,. \qquad (12)$$

The total elastic-scattering cross section can also be evaluated using the asymptotic expansions for the Bessel functions,

$$\sigma_{\mathrm{el}}=\pi\gamma^2\frac{\lambda}{\mu^2}\left[1+\frac{1}{8\lambda^2}+\cdots\right],\quad \lambda\gg 1\,. \qquad (13)$$

In this calculation, the integral over the scattering angle was converted into an integral over t, and the lower limit was extended from $-4p^2$ to $-\infty$. The resulting error is negligible. The ratio of σ_{el} to σ_T is readily obtained from Eqs. (12) and (13),

$$\sigma_{\mathrm{el}}/\sigma_T=\tfrac{1}{4}\gamma\left[1-\frac{1}{2\lambda}+\frac{1}{2\lambda^2}+\cdots\right],\quad \lambda\gg 1\,, \qquad (14)$$

and is clearly less than $\frac{1}{4}$ as is indicated by experiment.

This model for $f(s,t)$[21] was found to give an excellent fit to the p-p scattering data of Foley *et al.*[3,22] for incident momenta from 8.8 to 19.6 BeV/c. (Some difficulties were encountered at 6.8 BeV/c; these will be discussed later.) The parameters μ and λ in Eq. (11) were adjusted to fit the slope and curvature of the differential cross section $d\sigma_{\mathrm{el}}/dt$; γ was then adjusted to fit σ_T. Representative values of the parameters are given in Table I, and the associated differential cross sections are plotted in Figs. 1 and 2. It should be em-

TABLE I. Characteristics of the modified hyperbolic Bessel function fit to the p-p diffraction scattering data of Foley *et al.*[a] Except for $p_{\mathrm{lab}}=6.8$ BeV/c, the parameter λ was calculated using Eq. (15) for B^2 holding μ fixed at 7.7 m_π. For $p_{\mathrm{lab}}=6.8$ BeV c, μ was changed to 6.0 m_π and λ was again calculated from Eq. (15).

p_{lab} (BeV/c)	p (BeV/c)	σ_T (mb)	σ_{el} (mb)	γ	$B=(2\lambda/\mu^2)^{1/2}$ (F)
6.8	1.66	41.2	10.3	1.09	0.744
8.8	1.93	40.2	9.8	1.05	0.767
10.8	2.15	39.5	9.3	0.99	0.783
12.8	2.36	39.5	9.0	0.96	0.794
14.7	2.54	39.5	8.8	0.94	0.809
16.6	2.72	39.5	8.6	0.92	0.813
19.6	2.96	39.5	8.4	0.90	0.819
...	∞	39.5	6.6	0.66	0.968

[a] Ref. 3.

[19] We note only on possibility. If the absorptive interaction is sufficiently strong, the resultant suppression of the two-particle wave function in the interaction region leads to effects similar to those of a repulsive real potential, and the actual absorption in a given state may be significantly less than the maximum possible. The necessary assumption of an absorptive potential of increasing strength would be consistent with the requisite increase in the radius of the interaction region. However, arguments of this type, with their justification in potential scattering theory, may not be, and, in fact, probably are not, relevant to the present situation.

[20] G. N. Watson, Ref. 13, p. 416.

[21] We have considered a number of models other than that discussed in the text, and have been able to obtain reasonable fits to the data with several. All the successful models involved at least three parameters, and led, as would be expected, to weight functions $\mathrm{Im}H(b,s)$ which were practically identical for most values of b to that given in Eq. (9).

[22] We have used the interpolated data given in Fig. 4 of Ref. 3 in our analysis.

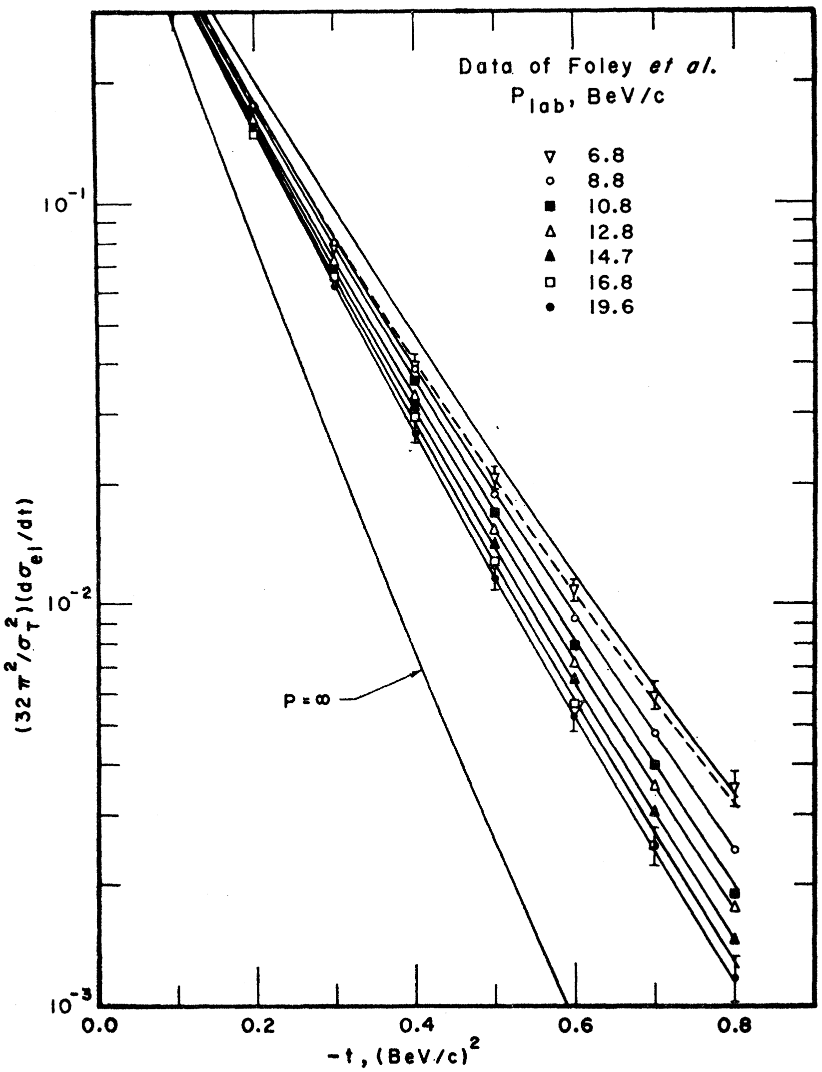

FIG. 1. Fit to the p-p differential scattering cross section $d\sigma_{el}/dt$ using the model of Sec. IIC. The data shown are the interpolated data of Foley *et al.*, given in Fig. 4 of Ref. 3. Both experimental and theoretical cross sections are normalized by division by the value of the forward cross section given by the optical theorem for a purely imaginary scattering amplitude. The solid and dashed curves for $p_{lab}=6.8$ BeV/c were calculated using B^2 from Eq. (15), and both the standard value and the modified value for μ, $\mu=7.7\ m_\pi$ and $\mu=6.0\ m_\pi$, respectively. Typical experimental errors are shown for the highest and lowest energies.

phasized that we have not attempted to obtain a best fit to the data of Foley *et al.*[3] nor have we taken into account the data of Diddens *et al.*[4] for values of $|t|$ beyond the upper limit of 0.8 (BeV/c)^2 in the experiment of Foley *et al.* [Fig. 2]. The differential cross section for $|t|>0.8$ (BeV/c)^2 is smaller than that for $t=0$ by factors of 5×10^{-3}–10^{-6}, and is consequently very sensitive to minor modifications in the first few partial-wave amplitudes. Reasonable fits to these data can accordingly be obtained by adding to the scattering amplitude as given in Eq. (11) very small terms of very short range in b. Typical corrections are 10^{-3}–10^{-4} of the main term for $t=0$, have ranges of about 0.3 F, and disappear slowly with increasing s.[23] Because of the very short-range nature of these terms, and the large uncertainties in the data for $|t|>0.8$ (Bev/c)^2, we do not regard these fits as being especially meaningful.[24] A representative short-range correction is nevertheless sketched in Fig. 2.

The observed narrowing of the elastic diffraction peak requires that λ increase, or that μ decrease, with increasing values of s. Our results do not discriminate between these possibilities; the narrowing of the peak is related primarily to a systematic change in the overall slope of $d\sigma_{el}/dt$, a quantity determined by the radius parameter $B=(2\lambda/\mu^2)^{1/2}$. The total cross section is proportional to B^2, but the requisite increase in this quantity is offset by a decrease in the opacity parameter γ. The energy dependence of B^2 and γ cannot be determined uniquely because of the limited energy range of the present experiments. It is nevertheless interesting to note that an expression for B^2 of the form

$$B^2=B_0^2[1+(p_0/p)]^{-1}, \qquad (15)$$

[23] Because of the short-range nature of the necessary correction terms, and the diminution of their influence on the cross section roughly as s^{-1}, it is tempting to associate these terms with the real rather than the imaginary part of the scattering amplitude, and in particular, with scattering by a soft repulsive core. See especially Sec. IID, and the remarks of G. E. Brown, Ref. 7.

[24] A view different from ours in this respect is maintained by R. Serber, Ref. 7.

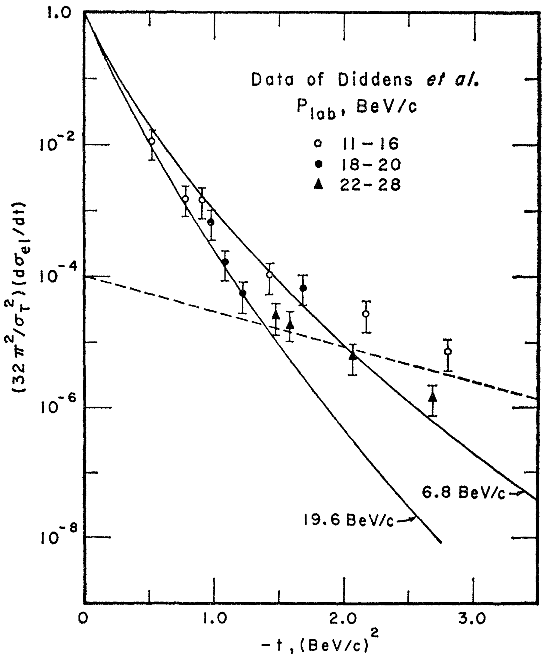

FIG. 2. Comparison of the predictions of the model of Sec. IIC for the p-p differential scattering cross section at 6.8 and 19.6 BeV/c, with the high $|t|$ data of Diddens *et al.* (Ref. 4). A possible exponential addition to the scattering amplitude which would bring the 19.6 BeV/c curve into agreement with the data is indicated. At $t=0$, this would represent a correction of relative magnitude 5×10^{-5} if added to the imaginary part of the scattering amplitude, or a real amplitude of magnitude 10^{-2} relative to the imaginary amplitude.

with $B_0^2=0.937$ F^2 and $p_0=1.15$ BeV/c, yields a remarkably good fit to the data.[25] Such a form is consistent with the expectation, based on the theory of high-energy potential scattering,[9,26] that $\text{Im}H(b,s)$ will differ from its asymptotic form at finite energies by terms of order p^{-1}. With the exception of the curve for $p_{\text{lab}}=6.8$ BeV/c, the curves in Fig. 1 were calculated using this expression for B^2 and assuming a fixed value of μ, $\mu=7.72m_\pi$ that is, a fixed long-range structure for $\text{Im}H(b,s)$. Should Eq. (15) be approximately correct for large p, the diffraction peak would eventually cease to shrink, and the elastic and total scattering cross sections would approach a constant ratio. As a curiosity, the asymptotic limits are indicated in Fig. 1 and Table I, but these are hardly to be taken seriously without some further corroboration of the model.

Despite its evident success in fitting the shape of the differential cross section $d\sigma_{\text{el}}/dt$, the foregoing model does not provide a satisfactory fit to σ_T at the lower momenta. The problem is evident from the values of the opacity parameter given in Table I. According to Eqs. (5), (6), and (9), this parameter is limited by unitarity and the assumption of a purely imaginary scattering amplitude to the range $0\leq\gamma\lesssim1$, the upper limit corresponding to total absorption of the low partial waves. [The approximations in Eqs. (5) and (6) do not cause any difficulty.] The violation of this upper limit for $p_{\text{lab}}=8.8$ BeV/c may be attributable to the 10% uncertainty in the absolute normalization of the experimental cross sections,[3,4] but this is perhaps a less likely explanation of the discrepancy for $p_{\text{lab}}=6.8$ BeV/c. What is more serious is the systematic trend exhibited in Table I, on the basis of which the difficulty would be expected to become much worse at lower energies; this is, in fact, the case, as may be seen from the data of Fujii *et al.*[27] at 2.1 and 2.9 BeV. Accepting the discrepancy as real, we are faced with the problem of altering the model so as to increase σ_T without at the same time disrupting the fit to $d\sigma_{\text{el}}/dt$. This is apparently not possible if we continue to ignore the spin dependence of $f(s,t)$, and maintain the restriction that $|\text{Re}f(s,t)|\ll|\text{Im}f(s,t)|$. The root of the difficulty may be seen from Eq. (3). The optical theorem provides a sum rule for the function $b\,\text{Im}H(b,s)$, and to increase σ_T, we must increase the area under this curve. Since the magnitude of $\text{Im}H(b,s)$ is bounded by the unitarity limit, the effective radius in b of this function must be increased, yet this radius, and the shape of the function, cannot be changed materially without destroying the fit to $d\sigma_{\text{el}}/dt$. These remarks are reflected in Eq. (12) by the proportionality of the total cross section to the parameter B^2 which determines the general slope of $d\sigma_{\text{el}}/dt$. Roughly speaking, then, the limit $\gamma\lesssim1$ and the assumption of a purely imaginary scattering amplitude imply that σ_T is bounded from above by a number related to the over-all slope of $d\sigma_{\text{el}}/dt$. This bound is violated in the present model, and (apparently) in all other reasonable models which are consistent with all the data. The only viable alternative is to discard one or both of our previous assumptions concerning the spin dependence and the real part of the scattering amplitude. In the following section we will consider a model in which the spin dependence of $f(s,t)$ is still ignored, but in which the assumption that $|\text{Re}f(s,t)|\ll|\text{Im}f(s,t)|$ is abandoned. It is probable that equally good results could be obtained by introducing an appropriate spin dependence of the scattering amplitude, for example, spin flip terms which necessarily vanish for $t=0$, but could contribute significantly to $d\sigma_{\text{el}}/dt$ for $t<0$; but we have not examined this possibility in detail.

[25] An equally good fit is obtained with the form

$$B^2=B_0^2[1-(p_0'/p)],$$

but this clearly leads to difficulties for small p. On the other hand, if the main variation in B^2 is attributed to changes in λ, that is, to changes in the value of b at which $\text{Im}H(b,s)$ first departs significantly from the limit imposed by unitarity, and if μ is assumed to be constant, the result in Eq. (15) implies that $\text{Im}H(b,s)$ assumes the form appropriate to a Yukawa potential as $p\to0$.

[26] R. Glauber, *Lectures in Theoretical Physics* (Interscience Publishers, Inc., New York, 1958), Vol. I, p. 315.

[27] T. Fujii, G. B. Chadwick, G. B. Collins, P. J. Duke, N. C. Hien, M. A. R. Kemp, and F. Turkot, Phys. Rev. **128**, 1836 (1962).

D. Proton-Proton Scattering: Amplitudes with $\mathrm{Re}f(s,t)\neq 0$

The admission of scattering amplitudes with $\mathrm{Re}f(s,t)\neq 0$ permits a much greater, and partially unwanted, flexibility in the analysis of proton-proton scattering. It is interesting, in particular, to inquire if the observed shrinking of the p-p diffraction peak could not, in fact, be explained by the slow disappearance with increasing energy of such a real amplitude,[28] with the t dependence of both $\mathrm{Re}f(s,t)$ and $\mathrm{Im}f(s,t)$ fixed. The answer is affirmative. It is consequently not possible to conclude from the observed shrinking that a Regge pole mechanism is operative, or even that the effective radius of the absorptive region is increasing, without first establishing experimentally the behavior of $\mathrm{Re}f(s,t)$ for $t<0$. We wish especially to emphasize that the observation that $|\mathrm{Re}f(s,0)/\mathrm{Im}f(s,0)|^2\ll 1$, established using the optical theorem, is insufficient for this purpose.[29]

The foregoing explanation for the narrowing of the p-p diffraction peak at high energies requires two specific assumptions about $\mathrm{Re}f(s,t)$; first, that the magnitude of this function decrease relative to that of $\mathrm{Im}f(s,t)$ for increasing s and fixed t, and second, that the magnitude of $\mathrm{Re}f(s,t)$ decrease less rapidly than that of $\mathrm{Im}f(s,t)$ for increasing $|t|$. Some arguments can be advanced to support both assumptions. For example, the contribution to the scattering amplitude from the exchange of a single spin-zero particle approaches zero as p^{-1} for large p and fixed t.[30] In contrast, a constant total cross section requires, through the optical theorem, that $\mathrm{Im}f(s,t)$ increase linearly with p for $t=0$. This behavior probably persists for $t\lesssim 0$. [This example, while suggestive, is not especially convincing because of the well-known difficulties at high energies with interactions which involve the exchange of particles of spins of one or greater.[30]] Perhaps more important at high energies are the contributions of absorptive processes. The contributions to the imaginary part of the scattering amplitude at $t=0$ are related by the optical theorem to the partial cross sections for the corresponding processes, are therefore positive, and add coherently. On the other hand, there are no such restrictions on the contributions to the real part of the scattering amplitude. This function would, therefore, not be expected in general to increase as rapidly with p for $t\sim 0$ as does $\mathrm{Im}f(s,t)$. The increase in the magnitude of $\mathrm{Re}f(s,t)$ relative to that of $\mathrm{Im}f(s,t)$ at large values of $|t|$ requires essentially that the range in b of $\mathrm{Re}H(b,s)$ be smaller than that of $\mathrm{Im}H(b,s)$. In support of this assumption, we may note that the strongest components of the (elastic) nucleon-nucleon interaction as determined at lower energies (<500 MeV) are of rather short range. These components would be expected to give the largest relative contributions to $\mathrm{Re}f(s,t)$ at high energies.[7] In addition, it may be observed that the contributions to $\mathrm{Re}H(b,s)$ of a purely absorptive potential which decreases slowly with increasing particle separation, are of shorter range in b than the corresponding contributions to $\mathrm{Im}H(b,s)$.

Aside from these rather nebulous arguments, there is little to guide us in the choice of the weight function $\mathrm{Re}H(b,s)$. This function need not have a fixed sign, and quite possibly passes through zero one or more times.[29] It is consequently difficult to argue that any specific model is reasonable, and we shall present only the results of a direct analysis of the data of Foley *et al.*[3,22] It was assumed that the t dependence of the functions $\mathrm{Re}f(s,t)$ and $\mathrm{Im}f(s,t)$ was the same at all energies, but that the magnitude of $\mathrm{Re}f(s,t)$ decreased relative to that of $\mathrm{Im}f(s,t)$ with the p^{-1} dependence on the center-of-mass momentum suggested by potential scattering considerations.[30a] The necessary functional forms for

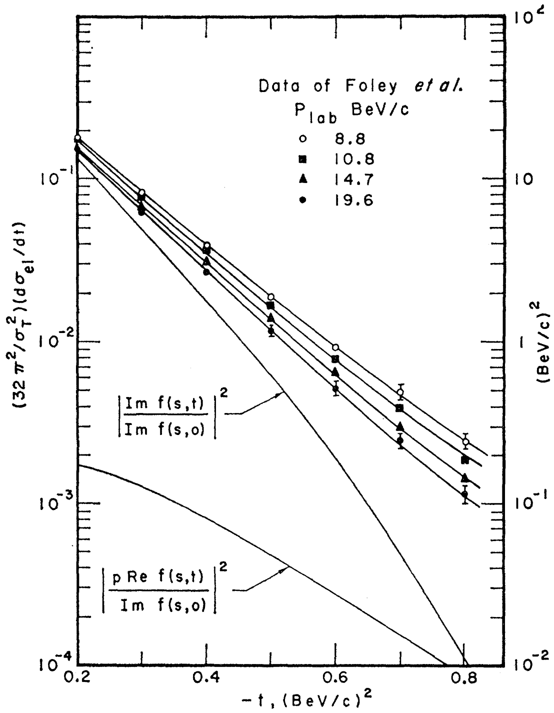

FIG. 3. Fit to the p-p differential cross sections of Ref. 3 at representative energies, obtained using the model of Sec. IID. The separate contributions to the cross section of the imaginary and real parts of the scattering amplitude are also shown; the right-hand scale should be used for the latter. Typical experimental errors are shown for the highest and lowest energies.

[28] See also the remarks in Ref. 23.

[29] For example, the real part of the scattering amplitude corresponding to the vacuum Regge pole vanishes at $t=0$, but is of significant magnitude for $t<0$, if either of the trajectories for $\alpha(t)$ suggested in Refs. 1 and 3 is correct.

[30] The exchange of a spin-zero particle gives the analog of nonrelativistic potential scattering by a velocity-independent Yukawa potential, treated in the first Born approximation. Velocity-dependent potentials lead to difficulties analogous to those encountered with the exchange of particles of spins 1 or greater.

[30a] *Note added in proof.* The indicated choice of a p^{-1} momentum dependence of $\mathrm{Re}f(s,t)$ relative to $\mathrm{Im}f(s,t)$ was based on considerations of nonrelativistic potential scattering using both real and

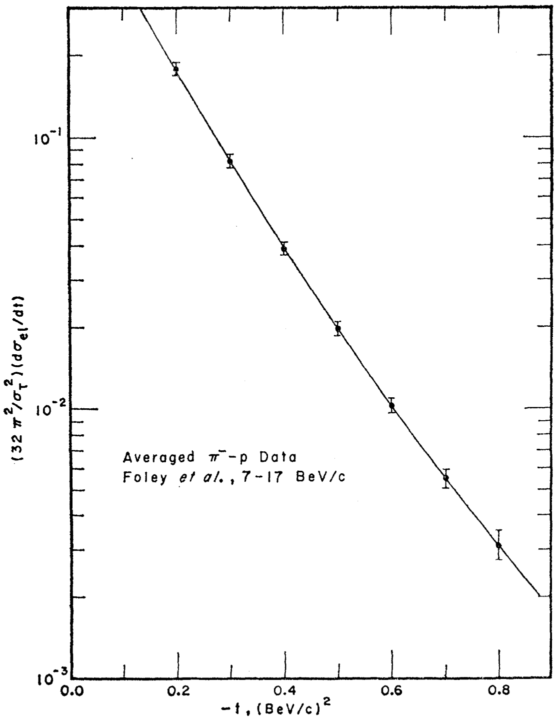

FIG. 4. Fit to the $\pi^- - p$ differential scattering cross section $d\sigma_{el}/dt$ using the model of Sec. IIC. Both experimental and theoretical cross sections are normalized by division by the value of the forward cross section given by the optical theorem for a purely imaginary scattering amplitude. The data points and the indicated errors are averaged values obtained from Fig. 5 of Foley *et al.* (Ref. 3).

$\mathrm{Re}f(s,t)$ and $\mathrm{Im}f(s,t)$ were then determined from the data. [However, the t dependence of the latter was restricted to be exponential for small t. It is probable that the fit could be improved by relaxing this restriction.] The results are shown in Fig. 3. We have again not attempted to find a best fit, but the over-all agreement with the data is generally good, even for this very specific assumption about the energy dependence. The 6.8 BeV/c data at small t are fit least well, but a very slight energy dependence in the form of either $\mathrm{Re}f(s,t)$ or $\mathrm{Im}f(s,t)$ would remedy the situation. We are, of course, not able to associate the energy-dependent contribution to the cross section unambiguously with the real part of the scattering amplitude; an appropriate energy dependence of the imaginary part could also be present. The precise behavior of $\mathrm{Re}f(s,t)$ can only be determined experimentally. This unfortunately appears to be rather difficult, since the real part of the amplitude is smallest relative to the imaginary part in the region near $t=0$ in which the simplest tests are available, namely Coulomb interference studies, and the comparison of the $t=0$ cross section with the lower bound given by the optical theorem. The present data do not extend to small enough values of t that we can use the model to make any definite predictions for such experiments. We note only that, if the apparent asymptotic form of $\mathrm{Re}f(s,t)$ is extrapolated to $t=0$ as a pure exponential, the predicted cross section exceeds that given by the optical theorem by only 10%. This may be a considerable overestimate, since the experimental curve is actually significantly below the asymptotic curve for $t=-0.2(\mathrm{BeV}/c)^2$; yet for large $|t|$, $|\mathrm{Re}f(s,t)|^2$ is several times as large as $|\mathrm{Im}f(s,t)|^2$.

imaginary potentials, the latter increasing in strength proportionally to p. The estimated relative momentum dependence would be changed to p^{-2} in the relativistic theory if $\mathrm{Re}f(s,t)$ was assumed to arise solely from the exchange of a single scalar or pseudoscalar particle. This is obviously unrealistic. However, the fit to the data would be improved by a stronger dependence of $\mathrm{Re}f(s,t)/\mathrm{Im}f(s,t)$ on p.

The foregoing model for elastic p-p scattering has several features which are attractive from the semiclassical point of view. First, the shrinking of the p-p diffraction peak can be explained by the expected diminution in the fractional contribution of $\mathrm{Re}f(s,t)$ to $d\sigma_{el}/dt$. The geometrical properties of the absorptive region [the b dependence of $\mathrm{Im}H(b,s)$] could therefore be fixed or slowly changing in the region of present interest. At higher energies, the diffraction peak would presumably cease to shrink. Second, the opacity of the interaction region would be essentially constant. A value $\gamma\approx 0.77$ is obtained by approximating the small t behavior of $\mathrm{Im}f(s,t)$ in Fig. 3 by an exponential, $\mathrm{Im}f(s,t)/\mathrm{Im}f(s,0)\approx\exp[5.2(\mathrm{BeV}/c)^{-2}t]$. The opacity could, in fact, increase slowly if the influence of the real phase shifts on the imaginary part of the scattering amplitude were taken into account [cf., Eq. (5)]. Finally, and basic to our motivation, this model is not subject to the difficulties with the opacity, or more generally, with unitarity and the optical theorem, which were discussed in the preceding section. Although the evidence for such a model is far from compelling, its attractive aspects, and the importance of understanding the relation between the shrinking [or nonshrinking] of diffraction peaks and the Regge pole mechanism, are such as to make additional experiments most desirable.

E. Analysis of Pion-Proton Scattering

We have used the model of Sec. IIC in an analysis of the recent data of Foley *et al.*[3] on elastic π^--p scattering. The total cross section for this process decreases very slowly over the range of the experiments,[31] 7.1–16.9 BeV/c laboratory momentum, and there is no significant change in the shape of the elastic diffraction peak. The scattering amplitude of Eq. (11) yields an excellent fit to the data. The results are shown in Fig. 4 for the parameters $\mu=6.8m_\pi$ and $\lambda=4.53$ [$B=0.74$ F]. Because of the slow decrease in the total cross section, the opacity parameter in this model must decrease; fit 2′ of Lindenbaum *et al.*[3,31] to $\sigma_T(\pi^- p)$ yields the relation

$$\gamma=0.64+0.65/p_{\mathrm{lab}}.$$

[31] S. J. Lindenbaum, W. A. Love, J. A. Niederer, S. Ozaki, J. J. Russell, and L. C. L. Yuan, Phys. Rev. Letters 7, 352 (1961).

However, the opacity could remain constant or increase over this energy range if the scattering amplitude had a small real part. The absence of any significant change in the shape of the diffraction peak would then require either that the effective radius parameters of $\mathrm{Re}H(b,s)$ and $\mathrm{Im}H(b,s)$ be nearly the same or that both change in a correlated fashion. However, the essentially complete absence of any structure in the diffraction peak,[32] and, to a lesser extent, the relatively large uncertainties in the total cross section, prevent more refined analysis. Without more detailed knowledge about the scattering amplitude, it appears unlikely that measurements of the elastic π-p cross section in the diffraction-scattering region can provide more than a rough consistency check for dynamical calculations. The same will of course be true of p-p scattering if the diffraction peak ultimately ceases to shrink, as suggested in the preceding sections. Experiments designed to detect any real part of the scattering amplitudes would consequently be of great interest.

[32] There may be some evidence of structure and shrinkage in the high $|t|$ region in the elastic $\pi^-$$-$$p$ cross sections at lower energies [Ref. 2]. It would be interesting to investigate this point in more detail.

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Pion-Proton Diffraction Scattering at Very Small Momentum Transfers*

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The model of high-energy scattering based on a moving pole and a fixed cut in the angular-momentum plane is applied to π-p scattering at very small momentum transfers, and is found to agree with the experiments which give a departure from an exponential diffraction peak.

IN a recent paper,[1] we have proposed a model of high-energy scattering, applicable to all strongly interacting particles, which is based on the assumption that the dominant singularities in the angular momentum plane are a Pomeranchuk pole and a fixed branch cut. For large values of the energy $(\sqrt{s})$ and small momentum transfers $[\sqrt{(-t)}]$ the cross section is given asymptotically by[2,3]

$$d\sigma/dt=(\pi/M_1{}^2M_2{}^2)|g(t)w^{\alpha_P(t)-1}+f(t)/\ln w|^2 \quad (1)$$

and

$$\sigma_T=(4\pi/M_1M_2)[g(0)+f(0)/\ln w],$$

where $w=(s/s_0)$, the dominant branch point $\alpha_2(t)=1$ for all t, and M_1, M_2 are the masses of the colliding particles. In the pole-fixed-cut model the normalization constant s_0 acquires special significance and taking $s_0=2M_1M_2$, this model explains both $p+p$ and π^-+p scattering data.[2,4,5] In the case of $p+p$ scattering w is relatively small in the range 7–30 BeV/c and the Pomeranchuk pole term in (1) dominates leading to a shrinkage of the diffraction peak, as given by a simple Regge pole. On the other hand, for π^-+p scattering between 7–17 BeV/c, w is several times larger and, for t not too close to zero, $w^{\alpha_P(t)-1}$ will be very small suppressing the pole term; the fixed cut then takes over and this gives a nonshrinking diffraction pattern, as is observed in π^-+p scattering. However, for t very small the pole should make a significant contribution and evidence of this is important because it establishes the presence of more than one significant singularity in the α plane.

Some evidence to this effect has recently been obtained from $\pi-p$ elastic scattering giving the differential cross section for very small values of t.[6] The differential cross section at these small momentum transfers deviates from the exponential shape usually adopted, exhibiting an upward curvature. This result indicates that there is an additional contribution at very small t over and above the fixed-cut contribution. Thus, the presence of at least two significant singu-

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[1] I. R. Gatland and J. W. Moffat, Phys. Rev. **129**, 2819 (1963).

[2] I. R. Gatland and J. W. Moffat, Phys. Rev. **132**, 441 (1963).

[3] The asymptotic form of the branch cut contribution was obtained in Ref. (1) by integrating by parts and dropping terms of higher order in $(1/\ln s)$. This calculation is exact for a discontinuity of the form $f(t,\alpha)=\theta(\alpha_2-\alpha)$. Such a discontinuity is, of course, unphysical, and in a less phenomenological treatment one might obtain $f(t)/(\ln s)^\beta$, say, with $\beta>1$ and noninteger. Due care should then be taken to avoid fixed cuts which cause the amplitude to become unbounded. However, the approximate analyses we have made are clearly insensitive to such a change, and the experimental features remain unaltered.

[4] K. J. Foley, S. J. Lindenbaum, W. A. Love, S. Ozaki, J. J. Russell, and L. C. L. Yuan, Phys. Rev. Letters **10**, 376 (1963).

[5] C. C. Ting, L. W. Jones, and M. L. Perl, Phys. Rev. Letters **9**, 468 (1962).

[6] S. Brandt, V. T. Cocconi, P. Fleury, G. Kayas, C. Pelletier, D. R. O. Morrison, F. Muller, and A. Wroblewski, Phys. Rev. Letters **10**, 413 (1963).